New theory of Lorentz violation from a general principle

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Abstract   We report that a general principle of physical independence of mathematical background manifolds brings a replacement of common derivative operators by co-derivative ones. Then we obtain a new Lagrangian for the ordinary minimal standard model with supplementary terms containing the Lorentz invariance violation information measured by a new matrix, denoted as the Lorentz invariance violation matrix. We thus provide a new fundamental theory to study Lorentz invariance violation effects consistently and systematically.

Key words   Lorentz invariance violation, physical independence, model beyond the standard model

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1   Introduction

Lorentz symmetry is one of the most significant and fundamental principles in physics, and it contains two aspects, namely Lorentz covariance and Lorentz invariance. Currently, there is an increasing interest in Lorentz invariance Violation (LV or LIV) both theoretically and experimentally [1]. We report here a fundamental theory to describe the LV effects based on a basic principle. Similar to the principle of relativity, which requires that the equations describing the laws of physics have the same form in all admissible frames of reference, we propose a physical independence principle that the equations describing the laws of physics have the same form in all admissible mathematical manifolds. We show that such a principle leads to the following replacement of the ordinary partial ∂_α and the covariant derivative D_α

$$\partial^\alpha \to M^{\alpha\beta}\partial_\beta, \quad D^\alpha \to M^{\alpha\beta}D_\beta, \qquad (1)$$

where $M^{\alpha\beta}$ is a local matrix. We first introduce the general principle, and then show that such principle leads to new supplementary terms violating Lorentz invariance in the Standard Model.

2   Principle of independence

Principle: Under any one-to-one transformation $X \to X' = f(X)$ of background mathematical manifolds, the corresponding transformation $\varphi(\cdot) \to \varphi'(\cdot)$ of an arbitrary physical field $\varphi(X)$ should satisfy

$$\varphi'(X') = \varphi(X). \qquad (2)$$

This statement actually makes the field $\varphi(X)$ represent a physical distribution rather than a mathematical function. A unique reality can be described in many ways $\varphi(X), \varphi'(X'), \varphi''(X''),\dots$ mathematically, but the physical essence remains unchanged, saying independence or invariance of mathematical descriptions. So (2) just claims Physical Independence or Physical Invariance (PI) of mathematical background manifolds. What we do here is just to put a physical requirement on a mathematical expression $\varphi(X)$.

For a given field $\varphi(X)$ satisfying (2), its derivative field is ordinarily defined as

$$\pi(X) = \partial_X \varphi(X). \qquad (3)$$

When $\pi(X)$ is a physical field, we should require the condition $\pi'(X') = \pi(X)$ according to PI. Therefore we need to check whether the definition (3) of the

derivative field satisfies (2). One easily finds

$$
\begin{aligned}
\pi(X) &= \partial_X \varphi(X) = \partial_X \varphi'(X') \\
&= \partial_X f(X) * \partial_{X'} \varphi'(X') \\
&= F(\partial_{X'}) \varphi'(X'),
\end{aligned}
$$

in which $F(\cdot) = \partial_X f(X) * \cdot$, where $F(\cdot)$ is linear to "$\cdot$". From the definition (3), $\pi'(X') = \partial_{X'} \varphi'(X')$, we see that $\pi'(X') \neq \pi(X)$. So this definition of the derivative field $\pi(X)$ of a physical field $\varphi(X)$ does not satisfy PI. The reason is due to the derivative with respect to the manifold X. So we define the derivative field as

$$
\pi(X) = M(\partial_X) \varphi(X), \tag{4}
$$

which indicates that $M(\cdot)$ is local and has the transformation property

$$
M(\cdot) \rightarrow M'(\cdot) = M(F(\cdot)), \tag{5}
$$

thus we have

$$
\begin{aligned}
\pi(X) &= M(\partial_X) \varphi(X) \\
&= M(\partial_X f(X) * \partial_{X'}) \varphi'(X') \\
&= M(F(\partial_{X'})) \varphi'(X') \\
&= M'(\partial_{X'}) \varphi'(X') \\
&= \pi'(X').
\end{aligned}
$$

According to (2), $\pi(X)$ is indeed a physical field with the new definition (4). The covariant derivative D_X has the same problem as ∂_X and can be handled in a similar manner. Thus we obtain our replacement for the ordinary ∂_X and the covariant derivative D_X by

$$
\partial_X \rightarrow M(\partial_X), \quad D_X \rightarrow M(D_X),
$$

whose explicit matrix form is

$$
\partial^K \rightarrow M^{KJ} \partial_J, \quad D^K \rightarrow M^{KJ} D_J.
$$

We need to point out that the above derivation is handled in Geometric Algebra $\mathcal{G}$ (or Clifford Algebra) and Geometric Calculus (see, e.g., Refs. [2, 3]), where the general element is called a multivector. Addition and various products of two multivectors are still a multivector, i.e., Geometric Algebra is closed. Different variables in physics, such as scalar, vector, tensor, spinor, twistor, matrix, etc., can be described by the corresponding types of multivectors in a unified form in Geometric Algebra (see Ref. [4] for a more detailed argument). So $\varphi(X) \in \mathcal{G}$ is a multivector-valued function of a multivector variable $X \in \mathcal{G}$. In the following

discussions, when we consider the space-time, which is part of the general Geometric Algebra space, x is adopted instead of X, and the indices are denoted by α, β instead of K, J.

The result (1) is similar to the gauge idea by Yang and Mills [5] from a basic consideration. When a local symmetry is considered, one has to replace a common partial ∂_α with a covariant derivative D_α to retain the invariance of a Lagrangian under the local gauge transformation. Requiring the property (2) for an arbitrary field, we must introduce the local matrix $M^{\alpha\beta}$ to make the common ∂^α a physical co-derivative operator $M^{\alpha\beta} \partial_\beta$. The combination of the above two general considerations brings about the new covariant co-derivative operator $D^\alpha \rightarrow M^{\alpha\beta} D_\beta$, which is essential for the natural introduction of LV terms in the Standard Model.

3   Standard model supplement

With the above considerations, we focus on the physical implications and consequences from the new introduced co-derivative $M^{\alpha\beta} \partial_\beta$ and covariant co-derivative $M^{\alpha\beta} D_\beta$. The effective Lagrangian $\mathcal{L}_{\mathrm{SM}}$ of the minimal standard model is composed of four parts

$$
\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{HF}}, \tag{6}
$$

$$
\mathcal{L}_{\mathrm{G}} = -\frac{1}{4} F^{a\alpha\beta} F^a_{\alpha\beta}, \tag{7}
$$

$$
\mathcal{L}_{\mathrm{F}} = i\bar{\psi}\gamma^\alpha D_\alpha \psi, \tag{8}
$$

$$
\mathcal{L}_{\mathrm{H}} = (D^\alpha \phi)^\dagger D_\alpha \phi + V(\phi), \tag{9}
$$

where we omit the chiral differences, the summation of chirality and gauge scripts. ψ is the fermion field, ϕ is the Higgs field, and $V(\phi)$ is the Higgs self-interaction. $F^a_{\alpha\beta} = \partial_\alpha A^a_\beta - \partial_\beta A^a_\alpha - g f^{abc} A^b_\alpha A^c_\beta$, $D_\alpha = \partial_\alpha + ig A_\alpha$ and $A_\alpha = A^a_\alpha t^a$, with A^a_α being the gauge field. g is the coupling constant, and f^{abc} and t^a are the structure constants and generators of the corresponding gauge group respectively. $\mathcal{L}_{\mathrm{HF}}$ is the Yukawa coupling between the fermions and the Higgs field, and is not related to derivatives, thus it remains unchanged under the replacement (1).

Under (1) and a decomposition $M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}$ which will be discussed later, the Lagrangians in (7)-

(9) become

$$\begin{aligned}
\mathcal{L}_{\mathrm{G}} &= -\frac{1}{4}(M^{\alpha\mu}\partial_\mu A^{a\beta} - M^{\beta\mu}\partial_\mu A^{a\alpha} - gf^{abc}A^{b\alpha}A^{c\beta}) \\
&\quad \times (M_{\alpha\mu}\partial^\mu A^a_\beta - M_{\beta\mu}\partial^\mu A^a_\alpha - gf^{abc}A^b_\alpha A^c_\beta) \\
&= -\frac{1}{4}F^{a\alpha\beta}F^a_{\alpha\beta} + \mathcal{L}_{\mathrm{GV}}, \quad (10) \\
\mathcal{L}_{\mathrm{F}} &= i\bar{\psi}\gamma_\alpha M^{\alpha\beta}D_\beta\psi = i\bar{\psi}\gamma^\alpha D_\alpha\psi + \mathcal{L}_{\mathrm{FV}}, \quad (11) \\
\mathcal{L}_{\mathrm{H}} &= (M^{\alpha\mu}D_\mu\phi)^\dagger M_{\alpha\nu}D^\nu\phi + V(\phi) \\
&= (D^\alpha\phi)^\dagger D_\alpha\phi + V(\phi) + \mathcal{L}_{\mathrm{HV}}, \quad (12)
\end{aligned}$$

with $M^{\alpha\beta}$ being the real matrix to maintain the Lagrangian hermitian. The last three terms $\mathcal{L}_{\mathrm{GV}}$, $\mathcal{L}_{\mathrm{FV}}$ and $\mathcal{L}_{\mathrm{HV}}$ of the equations mentioned above are the supplementary terms for the ordinary Standard Model. The explicit forms of these terms are

$$\begin{aligned}
\mathcal{L}_{\mathrm{GV}} &= -\frac{1}{2}\Delta^{\alpha\beta}\Delta^{\mu\nu}(g_{\alpha\mu}\partial_\beta A^{a\rho}\partial_\nu A^a_\rho - \partial_\beta A^a_\mu \partial_\nu A^a_\alpha) \\
&\quad - F^a_{\mu\nu}\Delta^{\mu\alpha}\partial_\alpha A^{a\nu}, \quad (13) \\
\mathcal{L}_{\mathrm{FV}} &= i\Delta^{\alpha\beta}\bar{\psi}\gamma_\alpha\partial_\beta\psi - g\Delta^{\alpha\beta}\bar{\psi}\gamma_\alpha A_\beta\psi, \quad (14) \\
\mathcal{L}_{\mathrm{HV}} &= (g_{\alpha\mu}\Delta^{\alpha\beta}\Delta^{\mu\nu} + \Delta^{\beta\nu} + \Delta^{\nu\beta})(D_\beta\phi)^\dagger D_\nu\phi \quad (15)
\end{aligned}$$

Thus we obtain a new effective Lagrangian for the Standard Model with new supplementary terms (SMS), denoted by $\mathcal{L}_{\mathrm{SMS}}$

$$\begin{aligned}
\mathcal{L}_{\mathrm{SMS}} &= \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{LV}}, \quad (16) \\
\mathcal{L}_{\mathrm{LV}} &= \mathcal{L}_{\mathrm{GV}} + \mathcal{L}_{\mathrm{FV}} + \mathcal{L}_{\mathrm{HV}}, \quad (17)
\end{aligned}$$

where $\mathcal{L}_{\mathrm{SMS}}$ satisfies the Lorentz covariance $(\mathrm{SO}^+(1,3))$, the gauge symmetry invariance of $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ and invariance under the requirement of PI (2), under which $\mathcal{L}_{\mathrm{SM}}$ cannot remain unchanged in a general situation.

We can have a better view of SMS here. The elements of $M^{\alpha\beta}$ are mass dimensionless, natural for the sign of testifying the Lorentz invariance, and they are not global constants generally. All of the LV terms are expressed in $\mathcal{L}_{\mathrm{LV}}$, and the LV information [6] is measured by a single concise matrix $\Delta^{\alpha\beta}$, which is convenient for a systematic study of the LV effects. To determine whether the Lorentz invariance holds exactly (see (19)), further work is needed to analyze the effective Lagrangian (16) of QED, QCD and EW (ElectroWeak) fields, and more experiments are needed to determine the magnitude of the elements in the matrix $M^{\alpha\beta}$.

4   Lorentz violation matrix

Now, let us focus on the new local matrix $M^{\alpha\beta}$, of which the vacuum expectation value is used for the coupling constants in (13), (14) and (15). We divide $M^{\alpha\beta}$ into two parts

$$M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}, \quad (18)$$

with $g^{\alpha\beta}$ being the space-time metric. The remaining $\Delta^{\alpha\beta}$ contains the information to judge whether the Lorentz invariance is kept or not

$$\Delta^{\alpha\beta} = \begin{cases} 0 & \text{no LV}, \\ \to 0 & \text{small LV}, \\ \text{otherwise} & \text{large LV}. \end{cases} \quad (19)$$

So $\Delta^{\alpha\beta}$ represents to what degree the Lorentz invariance is exact, and we call it the Lorentz invariance Violation Matrix (LVM). Intuitively, the smaller the elements of $\Delta^{\alpha\beta}$ are, the better the physics law holds Lorentz invariant. In this way, the LVM is similar to the CKM matrix which signals generation mixing and CP violation [7, 8], and it signals LV.

Generally, $\Delta^{\alpha\beta}$ might be particle-type dependent. If we use the vacuum expectation value of $\Delta^{\alpha\beta}$ for the coupling constants in the corresponding effective Lagrangian, not all of the 16 degrees of freedom of $M^{\alpha\beta}$ are physical. For the derivative field $M(\partial_x)\varphi(x)$ (4) of an arbitrary given field, $\varphi(x)$ can be rescaled to absorb one of the 16 degrees of freedom so that only 15 are left. When more fields are involved, there is only one degree of freedom that can be reduced from a rescaling consideration for all fields. Thus for generality, we may keep all 16 degrees of freedom in $M^{\alpha\beta}$ for a specific particle in our study.

With the rapid development of laboratory experiments [9] and astronomical observations [10–16], there will be more and more ways to determine the LVM $\Delta^{\alpha\beta}$ phenomenologically. For example, we can get the dynamical equations of fields such as modified Maxwell equations, the Klein-Gordon equation and the Dirac equation as well as various dispersion relations from the effective Lagrangian. As a preliminary test for our construction, we consider the Dirac equation for the fermion $\psi(x)$ through the substitution (1) or the Lagrangian (11)

$$(i\gamma_\alpha M^{\alpha\beta}\partial_\beta - m)\psi(x) = 0. \quad (20)$$

Multiplying both sides by $(i\gamma_\alpha M^{\alpha\beta}\partial_\beta + m)$, we obtain

$$(g_{\alpha\mu}M^{\alpha\beta}M^{\mu\nu}\partial_\beta\partial_\nu + m^2)\psi(x) = 0, \quad (21)$$

which is also a Klein-Gordon equation. So the dispersion relation is

$$p^2 + g_{\alpha\mu}\Delta^{\alpha\beta}\Delta^{\mu\nu}p_\beta p_\nu + 2\Delta^{\alpha\beta}p_\alpha p_\beta = m^2, \qquad (22)$$

with the Fourier transformation $\psi(x) = \int \psi(p)e^{-ip\cdot x}dp$ and (18). The last two items of the left side of (22) contain LV information. It is an extension of the ordinary mass-energy relation $p^2 = m^2$. Systematic analysis of LV here can be offered by adopting the general expression for $\Delta^{\alpha\beta}$. A special case is

$$\Delta^{\alpha\beta} = \mathrm{diag}(0,\xi,\xi,\xi), \qquad (23)$$

where we consider first only the diagonals for simplicity. Then (22) and (23) give

$$\begin{aligned} E^2 &= (1-\delta)\vec{p}^2 + m^2, \qquad (24)\\ \delta &= -\xi^2 + 2\xi. \end{aligned}$$

The photopion production of the nucleon in the GZK cutoff observations gives an available energy threshold $E \approx 10^{19}$eV [17]. For the proton, it can be used here to determine the magnitude of the upper limit of ξ, which is 10^{-23} from a rough estimate [18]. The details of analysis, which are presented in [4], are omitted here.

From the Lagrangian for free photons

$$\begin{aligned} \mathcal{L}_{\mathrm{G}} &= -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} - F_{\mu\nu}\Delta^{\mu\alpha}\partial_\alpha A^\nu \\ &- \frac{1}{2}\Delta^{\alpha\beta}\Delta^{\mu\nu}(g_{\alpha\mu}\partial_\beta A^\rho \partial_\nu A_\rho - \partial_\beta A_\mu \partial_\nu A_\alpha)(25) \end{aligned}$$

we get the modified Maxwell equation (or motion equation)

$$\Pi^{\gamma\rho}A_\rho = 0, \qquad (26)$$

where $\Pi^{\gamma\rho}$ is also the inverse of the photon propagator

$$\begin{aligned} \Pi^{\gamma\rho} &= -g^{\gamma\rho}\partial^2 + \partial^\gamma\partial^\rho \\ &+ \Delta^{\gamma\alpha}\partial^\rho\partial_\alpha + \Delta^{\rho\alpha}\partial^\gamma\partial_\alpha + \Delta^{\gamma\beta}\Delta^{\rho\nu}\partial_\beta\partial_\nu \\ &- g^{\gamma\rho}(2\Delta^{\mu\alpha}\partial_\mu\partial_\alpha + g_{\alpha\mu}\Delta^{\alpha\beta}\Delta^{\mu\nu}\partial_\beta\partial_\nu). \quad (27) \end{aligned}$$

The term $\partial^\gamma\partial^\rho$ is symmetric for the indices γ and ρ. So are $\Delta^{\gamma\alpha}\partial^\rho\partial_\alpha$ and $\Delta^{\rho\alpha}\partial^\gamma\partial_\alpha$. Hence the above three terms can be omitted under the constraint of the Lorentz gauge condition $\partial^\alpha A_\alpha = 0$ for the gauge field. With the Fourier decomposition $A_\rho = \int dp A_\rho(p)e^{-ip\cdot x}$ and the Lorentz gauge condition we can re-write Eq. (26) as

$$\Pi^{\gamma\rho}(p)A_\rho(p) = 0,$$

where

$$\begin{aligned} \Pi^{\gamma\rho}(p) &= g^{\gamma\rho}(p^2 + g_{\alpha\mu}\Delta^{\alpha\beta}\Delta^{\mu\nu}p_\beta p_\nu + 2\Delta^{\alpha\beta}p_\alpha p_\beta) \\ &- \Delta^{\gamma\beta}\Delta^{\rho\nu}p_\beta p_\nu, \end{aligned}$$

which is the inverse of the free photon propagator in the momentum space. A general parametrization for p_α can be done with spherical coordinates, so p_α can be expressed as $(E, -|\vec{p}|\sin\theta\cos\phi, -|\vec{p}|\sin\theta\sin\phi, -|\vec{p}|\cos\theta)$, where the light speed constant $c = 1$ is adopted. We find that there is a zero eigenvalue and a corresponding eigenvector $A_\rho(p)$ for the matrix $\Pi^{\gamma\rho}(p)$. So the determinant must be zero for the existence of the solution $A_\rho(p)$

$$\det(\Pi^{\gamma\rho}(p)) = 0. \qquad (28)$$

Then we have the equation

$$\sum_{i=0}^{8}\lambda_i(\Delta^{\alpha\beta},\theta,\phi)E^i|\vec{p}|^{8-i} = 0.$$

The coefficient $\lambda_i(\Delta^{\alpha\beta},\theta,\phi)$ is a variable with respect to the LVM $\Delta^{\alpha\beta}$ and the angles θ and ϕ. So there are 8 real solutions for $E(|\vec{p}|)$ at most, and in general there are no analytical solutions to a general high order linear equation. But for some simple cases of the LVM $\Delta^{\alpha\beta}$, we expect some analytical solutions for E. Anyway, E can be solved formally as $E = f_i(\Delta^{\alpha\beta},\theta,\phi)|\vec{p}|$, for $i = 1\ldots N$, and $1 \le N \le 8$. $f_i(\Delta^{\alpha\beta},\theta,\phi)$ is a real variable and is independent of the momentum magnitude $|\vec{p}|$ because the photon is massless in the Lagrangian of Eq. (25). So the free photon velocity is

$$c_{\gamma i} \equiv \frac{dE}{d|\vec{p}|} = f_i(\Delta^{\alpha\beta},\theta,\phi), \quad \text{for } i = 1\ldots N, \quad 1 \le N \le 8, \tag{29}$$

which means: i) The free photon propagates in the space with at most 8 group velocities; ii) For each mode, the light speed $c_{\gamma i}$ might be azimuth dependent and not a constant. As we know, the light spreads with different group velocities for different directions in the anisotropic media in optics. By analogy, we may view the space-time as a kind of media intuitively. However, there are essential differences between the optical case and the photon case here, because all the consequences of the N modes and the light speed anisotropy are the result of the Lorentz invariance violation or the space-time anisotropy suggested by the new theory.

Phenomenologically, the one-way experiment [19–22] performed at the GRAAL facility of the European Synchrotron Radiation Facility (ESRF) in Grenoble, reported results on the light speed anisotropy by Compton scattering of high-energy electrons on laser photons. A detailed analysis, which will be given elsewhere [23], shows that the azimuthal distribution of the GRAAL data can be elegantly explained by our new theory.

5   Conclusion

In summary, with a general requirement of physical independence or physical invariance of mathematical background manifolds, we introduce a replacement of common derivative operators by co-derivative ones. This naturally brings about Lorentz invariance violation terms in the Standard Model, and we get a Lorentz invariance violation matrix $\Delta^{\alpha\beta}$, which can describe the Lorentz invariance violation effects in a systematic and consistent manner. The novel Lorentz invariance violation matrix in this article has the following merits: i) it is natural, because we introduce it under a general consideration which makes a field indeed a physical field without adding Lorentz invariance violation terms by hand; ii) it is systematic, since the information of Lorentz invariance violation can be extracted from it uniquely; iii) it is simple, since the Lagrangian $\mathcal{L}_{\text{SMS}}$ provides a fundamental framework for elegant applications to experimental problems. We thus provide a new fundamental theory to study the Lorentz invariance violation effects consistently and systematically.

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